Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

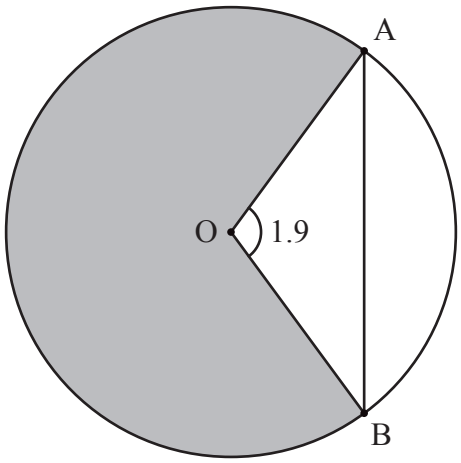
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\hat{O}B = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord $[AB]$. [3]

(b) Find the area of the shaded sector. [3]

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2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

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3. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $\$45\,000$, and her annual salary increases 2% on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary $(\$S)$	$45\,000$	$47\,200$	$48\,500$	$53\,000$	$57\,000$

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

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4. [Maximum mark: 6]

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$.

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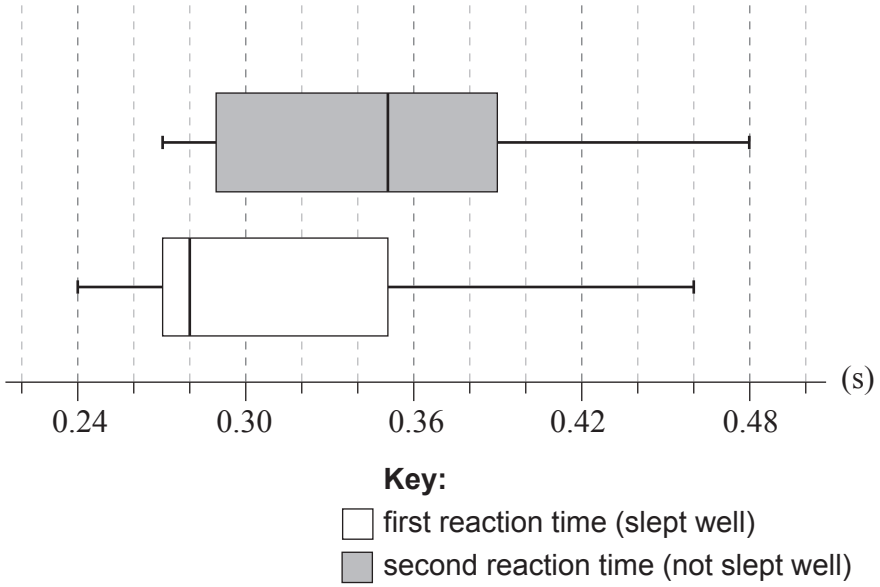
5. [Maximum mark: 6]

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

(a) State the median reaction time after sleeping well. [1]

(b) Verify that the measurement of 0.46 seconds is not an outlier. [3]

(c) State why it appears that the mean reaction time is greater than the median reaction time. [1]

Now consider the two box and whisker diagrams.

(d) Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time. [1]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v = \frac{\left(t^2+1\right)\cos t}{4}$, $0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\text{m s}^{-2}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

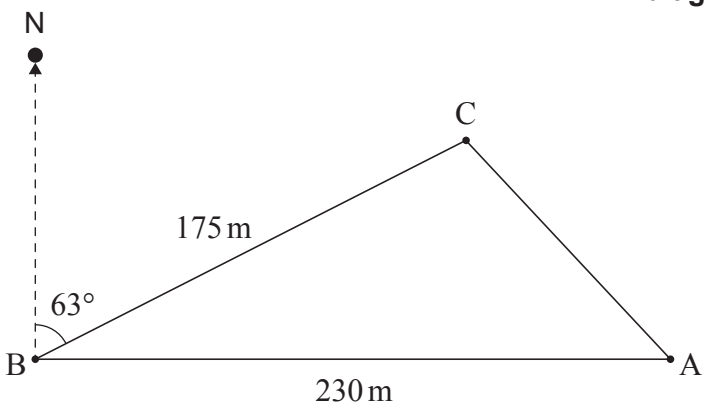
7. [Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of $063°$ to reach point C.

This is shown in the following diagram.

diagram not to scale



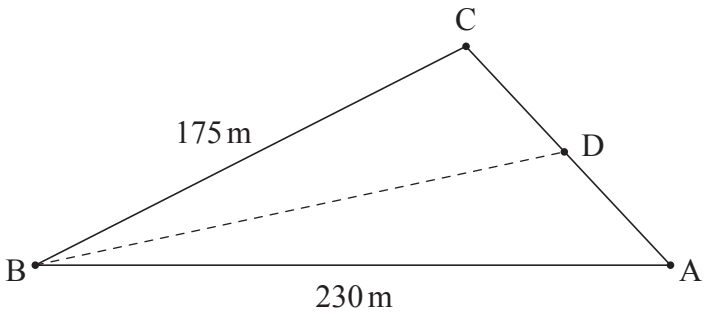
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $C\hat{A}B$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

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8. [Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, $h_A \text{cm}$, at time t weeks can be modelled by the function $h_A(t) = \sin(2t + 6) + 9t + 27$, where $0 \leq t \leq 9$.

The scientist found that the height of Plant B, $h_B \text{cm}$, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \leq t \leq 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $0 \leq t \leq 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 18]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

(a) Find the value of σ. [4]

(b) On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. [2]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

(c) Find the probability that she will be late to work at least one day next week. [3]

(d) Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. [5]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

(e) Find the probability that Suzi will receive a bonus. [4]

References:

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Answers written on this page
will not be marked.